**MATHEMATICS
APPLICATIONS
UNIT 3**
Section Two:
Calculator-assumed

SOLUTIONS

Student Number: In figures

In words   _____

Your name   _____

Time allowed for this section
Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction
fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper,
and up to three calculators approved for use in the WACE examinations

Important note to candidates
No other items may be taken into the examination room. It is **your** responsibility to ensure that
you do not have any unauthorised notes or other items of a non-personal nature in the
examination room. If you have any unauthorised material with you, hand it to the supervisor
before reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	97	65
			Total	149	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
    - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
    - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed 65% (97 Marks)

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

Question 8 (7 marks)

After the failure of a computer containing the details of 312 clients, Chris was given the job of re-entering all the client information into a new computer. On the first day he managed to re-enter the details of 15 clients. On each subsequent day, he was given more and more time for this job and managed to add two more clients each day than on the previous day.

(a) How many clients did Chris re-enter on the third day? (1 mark)

> 15, 17, 19: **19** clients.

(b) How many clients had Chris re-entered altogether after five days? (2 marks)

> $15 + 17 + 19 + 21 + 23 = 95$ clients

(c) Deduce a rule for T_n, the number of clients re-entered on day n in the form $T_n = an + b$.
(2 marks)

> $$T_n = 15 + (n-1) \times 2$$
> $$= 15 + 2n - 2$$
> $$= 2n + 13$$

(d) On which day did Chris finish the job, and how many clients did Chris re-enter on this day? (2 marks)

> *Using CAS*
>
> On day 12, when he re-entered the details of 37 clients.

$a_{n+1}=a_n+2$		
n	a_n	Σa_n
1	15	15
2	17	32
3	19	51
4	21	72
5	23	95
6	25	120
7	27	147
8	29	176
9	31	207
10	33	240
11	35	275
12	37	312
13	39	351

See next page

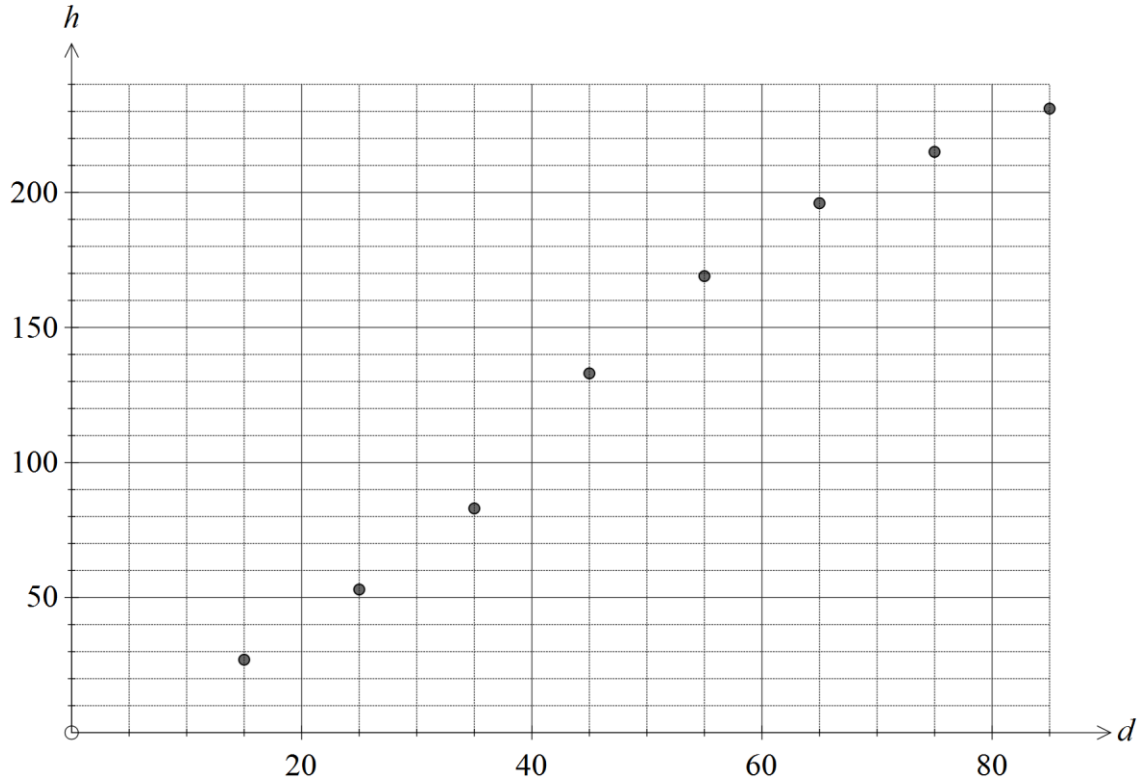**Question 9**   **(8 marks)**

The average height of 30 sunflower plants was recorded to the nearest centimetre in the table below over a period of 95 days.

Day (d)	15	25	35	45	55	65	75	85
Average height (h cm)	27	53	83	133	169	196	215	231

(a)   Plot these points on the axes below.   (2 marks)

(b)   Determine the correlation coefficient between d and h, and comment on the strength and direction of the association.   (2 marks)

> $r = 0.9896$
>
> Strong positive association.

(c)   State the equation of the least-squares line that models the relationship between these variables.   (2 marks)

> $h = 3.111d - 17.161$

(d)   Clearly explain what the value of the slope of the least-squares line means in the context of this question.   (2 marks)

> The average height of the plants is increasing at an average of 3.111 cm per day.

See next page

Question 10 **(8 marks)**

The recursive formula $T_{n+1} = 1.005T_n$, $T_0 = 2000$ is used to calculate the value of an initial deposit of $2 000 after n months when interest, at a rate of 6% per annum, is compounded monthly.

(a) Calculate the value of the deposit after

  (i) one month. (1 mark)

$$2000 \times 1.005 = \$2\ 010$$

  (ii) 12 months. (2 marks)

$$T_{12} = 2123.35562$$
$$\approx \$2123.36$$

(b) The formula $T_n = ar^n$ can also be used to calculate the value of the investment. State the values of the constants a and r. (2 marks)

$$a = 2000$$

$$r = 1.005$$

(c) Determine the total amount of interest that has accumulated on the deposit over the first 12 months. (1 mark)

$$2123.36 - 2000 = \$123.36$$

(d) Determine how many months it would take for $400 interest to accumulate. (2 marks)

$$T_n = 2400$$

$$T_{36} = 2393.36$$
$$T_{37} = 2405.33$$

37 months.

See next page

Question 11   **(9 marks)**

The table below shows the paired scores of twelve students in a Chemistry and a Physics exam, and the Chemistry score of a thirteenth student who was absent for the Physics exam.

Student	Chemistry score (C)	Physics score (P)
1	53	40
2	63	46
3	95	70
4	66	42
5	67	47
6	63	55
7	70	59
8	70	63
9	80	58
10	77	61
11	74	67
12	80	66
13	84	-

(a) Graph the data on your calculator and use features of the graph to explain why the Physics teacher thought that it would be reasonable to predict the Physics score of Student 13 from their Chemistry score.   (2 marks)

> The points display a straight line pattern, showing a moderate to strong positive linear association.

(b) Name the explanatory variable.   (1 mark)

> Chemistry score.

(c) Calculate the correlation coefficient r_{CP}.   (1 mark)

> 0.812

(d) Determine what percentage of the variation in the Physics scores can be explained by the variation in the Chemistry scores.   (2 marks)

> $r^2 = 0.812^2 = 0.659$
>
> 66% of the variation

(e) Determine the equation of the least-squares line that models the relationship between the Chemistry and Physics scores, rounding coefficients to two decimal places.   (2 marks)

> $P = 0.78C + 1.32$

(f) Predict the Physics score of Student 13.   (1 mark)

> $0.78(84) + 1.32 = 66.84 \approx 67$

See next page

Question 12 (8 marks)

A delivery driver based at a depot, D, has to deliver parcels to three locations A, B and C and then return to D. The times to travel between each of the locations in minutes is shown below.

Travel time (mins)	A	B	C	D
A	-	9	8	11
B	9	-	13	12
C	8	13	-	10
D	11	12	10	-

(a) The driver decides to travel from D to A to B to C and back to D. Calculate how long this route would take. (2 marks)

$$11+9+13+10 = 43 \text{ minutes}$$

(b) Construct a graph to show this information. (2 marks)
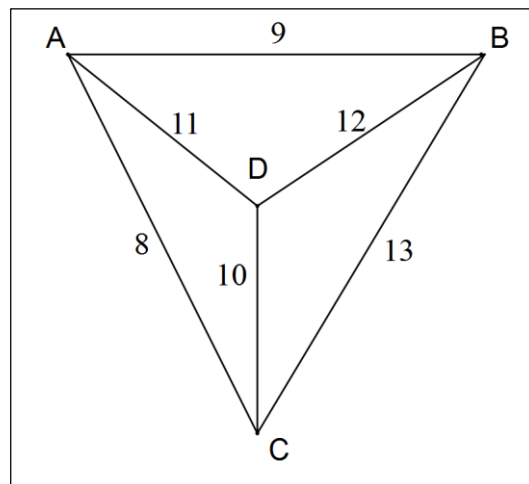
(c) Determine the route the driver should take to minimise the travelling time and state what this time is. (2 marks)

D-B-A-C-D, which takes 39 minutes.

(d) If heavy traffic after a sports event caused the times to travel from D to B and from D to C to both increase by 10 minutes, state the effect, if any, that these changes have on your answer to (c). (2 marks)

Route changes to D-A-B-C-A-D and now takes 52 minutes.

See next page

Question 13 **(7 marks)**

(a) The proportional bar graph below shows the preferred hot beverage chosen by arts and science undergraduates.

Preferred hot beverage
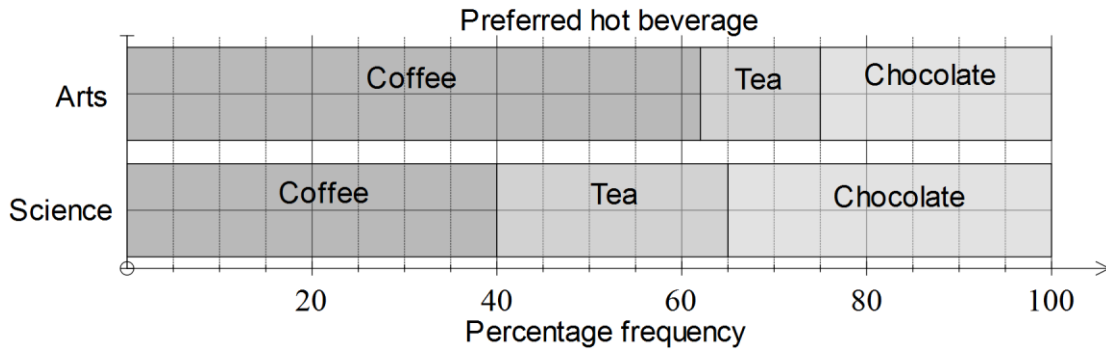
Discuss whether any association exists between the preferred hot beverage and type of undergraduate. (2 marks)

> Yes, association exists.
>
> Proportions preferring each type of drink are seen to be markedly different for arts and science undergraduates.

(b) A random survey of 328 voters who lived in the suburbs Rossmont and Clarecross were asked which one of the political parties - Labour, Liberal or Green - they supported. The results are shown in the table below.

	Labour	Liberal	Green	Other
Rossmont	36	56	23	29
Clarecross	45	78	27	35

(i) How many more voters were surveyed in Clarecross than Rossmont? (1 mark)

> $185 - 144 = 41$ **voters**

(ii) Construct a percentage two-way table that will help identify whether political party support is associated with suburb. (3 marks)

	Labour	Liberal	Green	Other	*Total*
Rossmont	25	39	16	20	*100*
Clarecross	24	42	15	19	*100*

(iii) Comment on whether political party support appears associated with suburb.
(1 mark)

> No evidence, as support for various parties similar in each suburb.
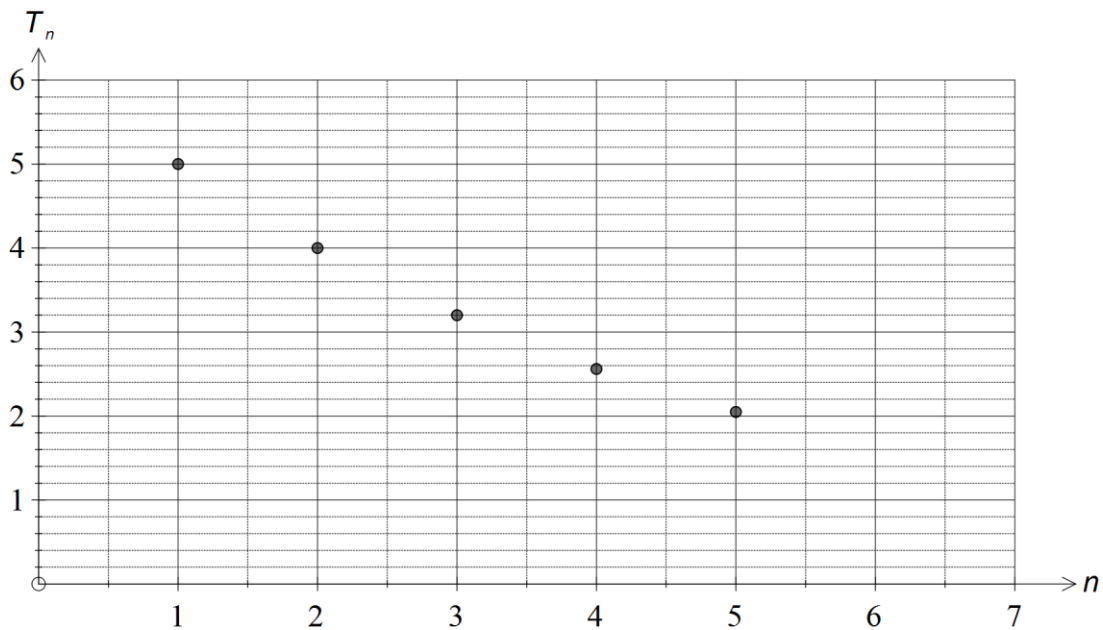
See next page

Question 14 **(8 marks)**

Sequence A is defined given by $T_{n+1} = 0.8T_n$, $T_1 = 5$.

(a) Use the rule to complete the first five terms of Sequence A in the table below. (2 marks)

n	1	2	3	4	5
T_n	5	**4**	**3.2**	**2.56**	**2.048**

(b) Graph the first five terms of sequence A on the axes below. (2 marks)

(c) How many terms of Sequence A are greater than 1? (1 mark)

> 8 terms.
> ($T_9 = 0.8...$)

(d) The terms of the sequence can also represent the value of a secondhand car (in thousands of dollars) at the start of each year (year n).

(i) Determine the value of the car at the start of the sixth year. (1 mark)

> $T_6 = 1.6384$ ⇒ value is $1638.40

(ii) By what percentage is the value of the car decreasing each year? (1 mark)
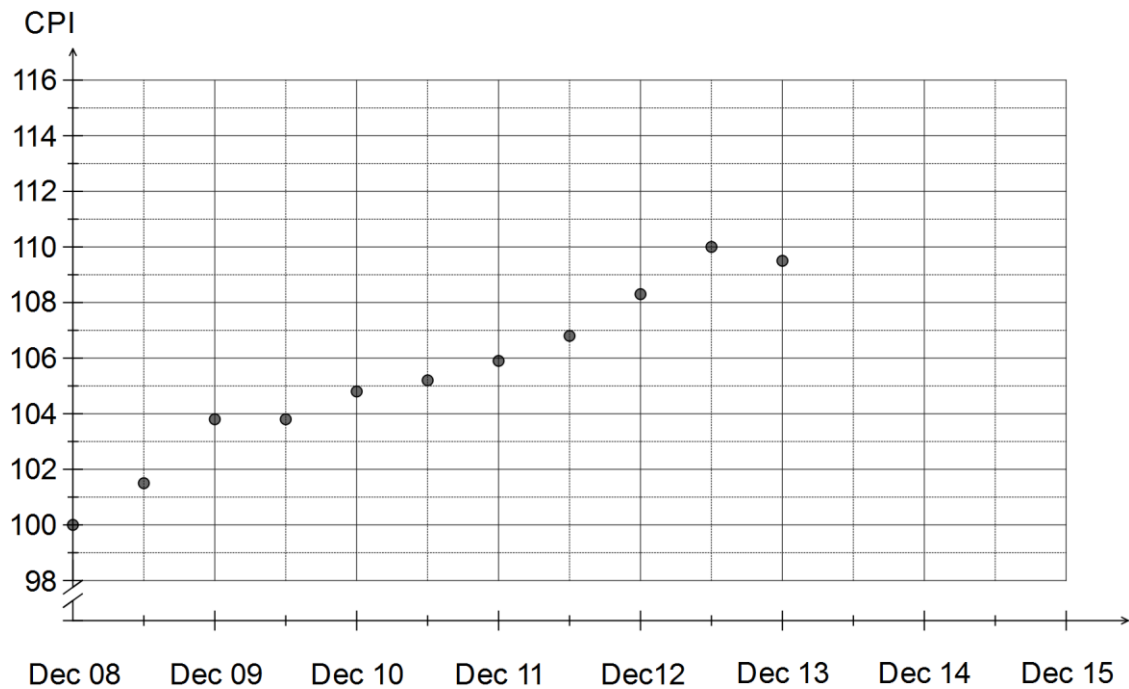
> 20%

(iii) The value of the car is written off when it falls below $500. At the start of which year will this occur? (1 mark)

> Year 12. (Value has fallen to $429...)

See next page

Question 15   (8 marks)

The graph below shows the consumer price index (CPI) at six monthly intervals from December 2008 until December 2012.


(a)   The consumer price index values for June and December 2013 were 110 and 109.5 respectively. Add these two points to the graph above.   (1 mark)

The equation of the least-squares line for all 11 pairs of points is $y = 0.928x + 100.78$, where x is the number of half-years since December 2008 (for example, for December 2009, $x = 2$) and y is the consumer price index.

(b)   Calculate the residuals for June and December 2013.   (2 marks)

June 2013

$0.928(9) + 100.78 = 109.13$
$110 - 109.13 = 0.87$

Dec 2013

$0.928(10) + 100.78 = 110.06$
$109.5 - 110.06 = -0.56$

See next page

The residual plot for the first nine data points is shown below.
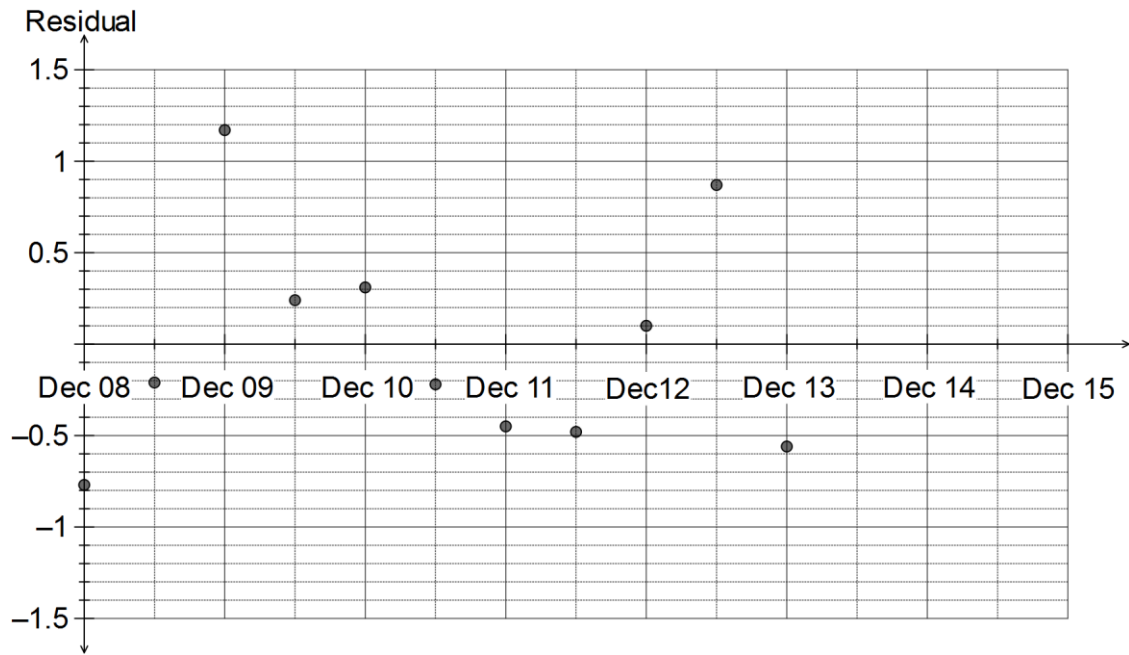
Residual

(c)   Add the residuals from (b) to the graph.   (1 mark)

(d)   Use the residual plot to comment on the appropriateness of fitting a linear model to the
        data.   (1 mark)

> Linear model is appropriate as no pattern evident in residuals.

(e)   Estimate the consumer price index for

        (i)   June 2014.   (1 mark)

        $$0.928(11) + 100.78 = 110.988 \approx 111.0$$

   (ii)   December 2015.   (1 mark)

        $$0.928(14) + 100.78 = 113.772 \approx 113.8$$

(f)   Which of the two estimates in (e) is the most reliable? Justify your answer.   (1 mark)

> June 2014, as this prediction involves the least amount of
> extrapolation and so is more reliable.

See next page

Question 16   **(10 marks)**

The digraph below represents the bus services available between four transport hubs -
an airport (A), two train stations (B and C) and a ferry terminal (D).
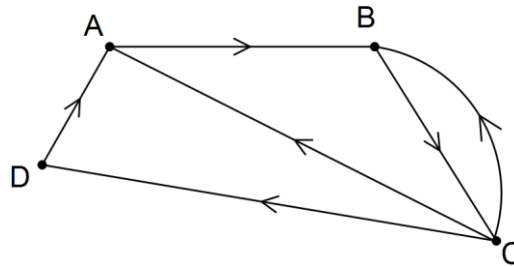
Reachability is the ability to reach one vertex from another along a walk of length one or more in a graph.

(a)   State how many ways, if any, the airport A is reachable from train station C along a walk

     (i)   of length 1.   (1 mark)

> 1 way *(CA)*

     (ii)   of length 3.   (1 mark)

> 1 way *(CBCA)*

(b)   Construct the adjacency matrix, M, for the digraph.   (3 marks)

$$
\begin{array}{c}
\text{To}
\end{array}
$$

		A	B	C	D
	A	0	1	0	0
From	B	0	0	1	0
	C	1	1	0	1
	D	1	0	0	0

See next page

(c) Calculate M^2 and use it to determine how many different walks of length two exist in the digraph. (2 marks)

$$\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Eight different walks.
(Sum of matrix coefficients)

(d) Calculate $M + M^2$. (1 mark)

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \\ 2 & 2 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

(e) B can be reached from A using just one bus service and C can be reached from A using two bus services. Some of the transport hubs, however, are not reachable from a different hub using two or fewer bus services. List all such pairs of hubs. (2 marks)

D not reachable from A

C not reachable from D

(Zeros not on diagonal in (d))

Question 17   **(10 marks)**

As part of some research into how well people were able to estimate the fat content of some common foods, a large group of people were randomly selected to view 11 foodstuffs and write down what weight of fat the foodstuffs contained. The actual (x) and average estimate calculated from the group (y) for the foodstuffs are shown in the table below.

Foodstuff	Fat content of foodstuff, in grams	
	Actual (x)	Group Estimate (y)
Ranch dressing	5	3
Candy bar	18	16
Hot dog	12	13
Chicken sandwich	25	24
Cheeseburger	22	21
2 pieces fried chicken	40	38
Nachos with cheese	19	20
Doughnut	14	12
Chicken nuggets	13	11
Churro, 1 stick	6	16
Vegetable fried rice	10	7

This data is shown on the scatterplot below.
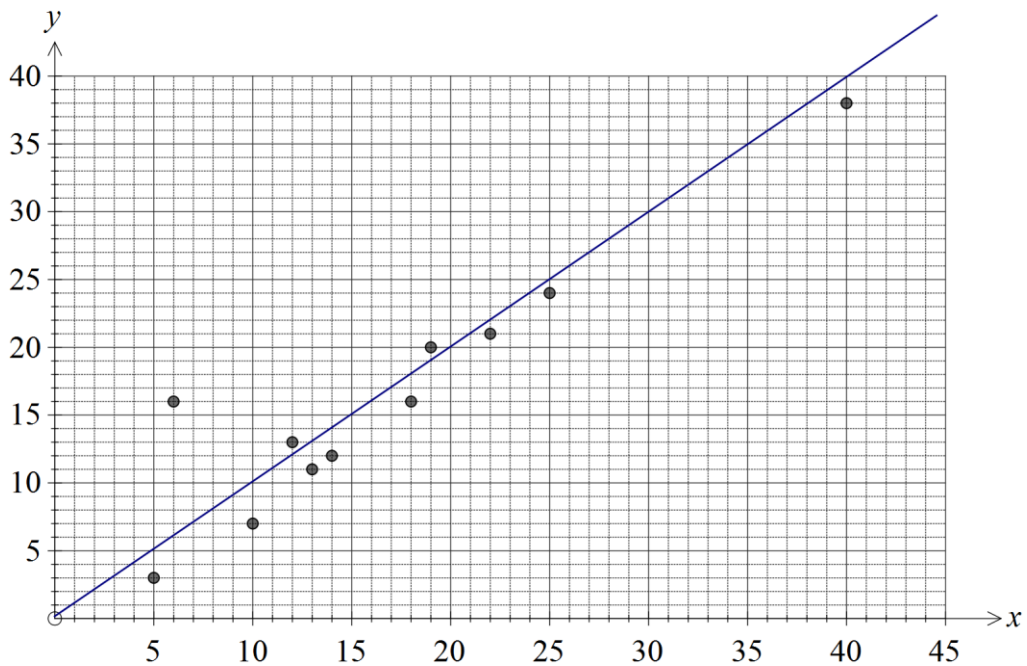
(a)   On the graph, draw the line $y = x$ and use it to explain whether the group tended to over or under estimate the fat content compared to the actual fat content.   (2 marks)

> Tended to under estimate, as there are more points below the line (8) than above (3).

See next page

(b) Calculate the correlation coefficient, and use it to comment on the strength of the association between the groups' estimated fat content and the actual fat content.

(2 marks)

$r = 0.931$

The closeness of r to 1 indicates a strong association between the estimated and the actual fat content.

(c) For one foodstuff, the ability of the group to estimate the fat content was considerably different to the others.

    (i) Which foodstuff was this? (1 mark)

Churro, 1 stick

    (ii) Remove this point from the dataset, recalculate the correlation coefficient and explain any change in its value. (2 marks)

$r = 0.991$

It has increased and become closer to 1 because the remaining points all lie closer to a straight line.

(d) One foodstuff on the scatter plot is not close to any others.

    (i) Which foodstuff was this? (1 mark)

2 pieces fried chicken

    (ii) Also remove this point from the dataset, recalculate the correlation coefficient for the remaining 9 foodstuffs and explain any change in its value. (2 marks)

$r = 0.981$

It has decreased slightly, because the point was very close to the regression line and so removing it has increased the average spread of the remaining points about the new line.

See next page

Question 18   **(7 marks)**

The temperature, T_n in °C, inside a refrigerated room n hours after being switched on can be modelled by the rule $T_{n+1} = 0.7T_n - 3.6$, $T_0 = 15$.
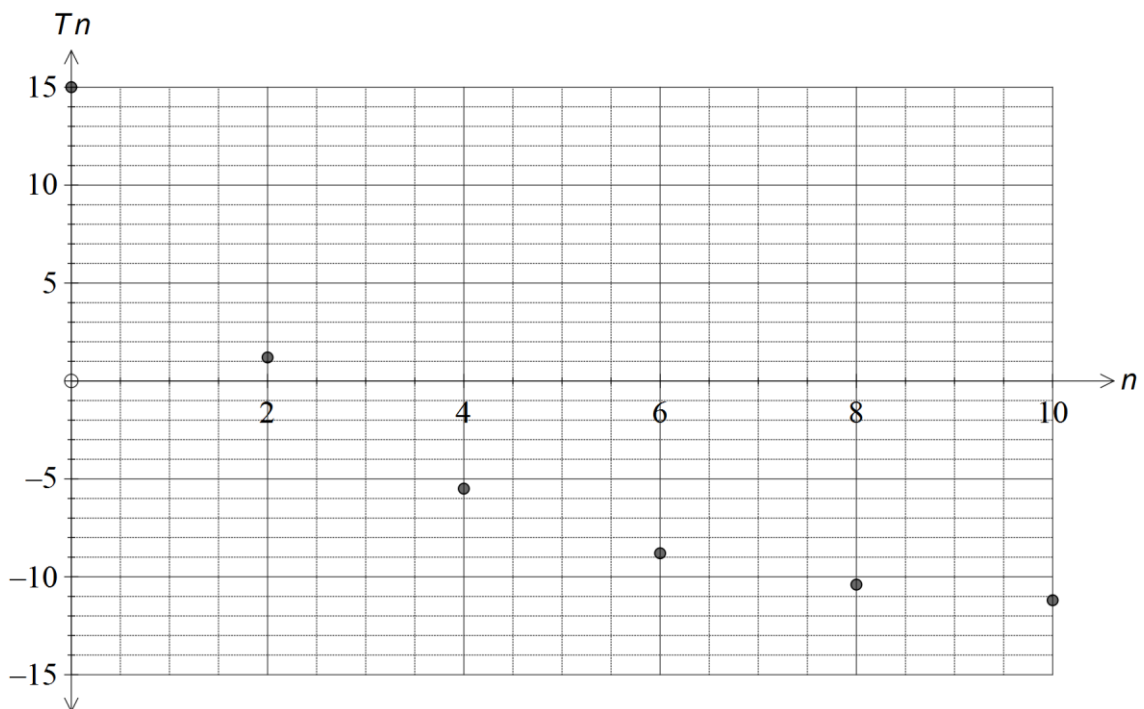
(a)   What was the initial temperature inside the room, just as the cooling system was turned on?   (1 mark)

> 15°C

(b)   Complete the missing values in table below, rounding to one decimal place.   (2 marks)

n	0	2	4	6	8	10
T_n	**15.0**	**1.2**	**-5.5**	**-8.8**	-10.4	-11.2

(c)   Plot the points in the table on the axes below.   (2 marks)

(d)   By finding the steady state solution of the recurrence relation, or otherwise, determine the temperature that the refrigerated room will fall to when the cooling system is left on for a long time.   (2 marks)
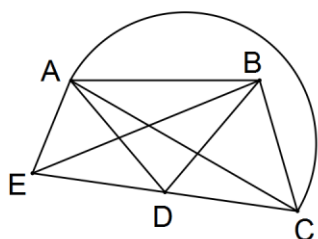
> $T = 0.7T - 3.6 \implies T = -12°C$.
>
> *(Solve using CAS)*
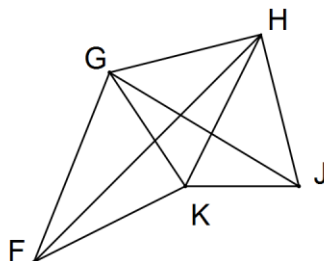
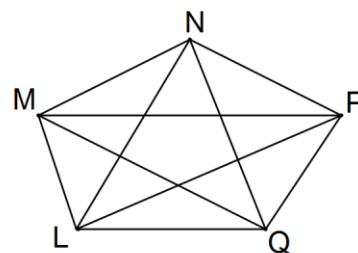**See next page**

Question 19 (7 marks)

Three connected graphs, each with five vertices, are shown below.

Graph I   Graph II   Graph III

(a) State which of the graphs above are planar and explain your choice(s). (2 marks)

> Graph II is the only planar graph, as it is the only one that can be redrawn so that no edges cross.

(b) Construct an adjacency matrix for Graph I. (3 marks)

$$
\begin{array}{c c c c c c}
 & A & B & C & D & E \\
A & 0 & 1 & 1 & 1 & 1 \\
B & 1 & 0 & 1 & 1 & 1 \\
C & 1 & 1 & 0 & 1 & 0 \\
D & 1 & 1 & 1 & 0 & 1 \\
E & 1 & 1 & 0 & 1 & 0
\end{array}
$$

(c) Determine the number of faces on the undirected planar graph with six vertices that has the adjacency matrix shown: (2 marks)

$$
\begin{array}{c c c c c c c}
 & A & B & C & D & E & F \\
A & 0 & 1 & 1 & 1 & 1 & 1 \\
B & 1 & 0 & 1 & 1 & 1 & 1 \\
C & 1 & 1 & 0 & 1 & 1 & 0 \\
D & 1 & 1 & 1 & 0 & 0 & 0 \\
E & 1 & 1 & 1 & 0 & 0 & 1 \\
F & 1 & 1 & 0 & 0 & 1 & 0
\end{array}
$$

> Sum of coefficients + sum of leading diagonal = 24 = 2 x edges, so edges = 12.
> Planar, so using Euler, $6 + f = 12 + 2 \implies f = 8$.
> Graph has 8 faces.

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____